ABSTRACT

For improving the reliability and efficiency of the dynamic modelling and simulation of geneva mechanism, the corresponding vector bond graph procedure is proposed. According to the kinematic relations, the vector bond graph model of point-follower is made. Based on this, the vector bond graph model of geneva mechanism can be made. For the difficulties brought by differential causality in the system automatic modeling and simulation, the effective bond graph augment method is proposed. As a result, the automatic modelling and simulation of a geneva mechanism on a computer is realized by corresponding algorithm. This procedure is very suitable for the modeling and simulation of the systems containing multi-energy domains in a unified manner, its validity is illustrated by a practical example.

Key words: Vector bond graph; Modelling and simulation; Geneva mechanism; Bond graph augment; Causality

INTRODUCTION

As an indexing mechanism, geneva mechanism is widely used in modern automatic machine. For improving system dynamic character, system modeling and simulation are very important. For improving the reliability and efficiency of the dynamic modeling and simulation of complex multibody systems, e.g. the geneva mechanism or geneva connecting-rod combination mechanism, different procedures have been proposed in previous work[1,2]. The Newton-Euler technique and Lagrange technique are two of the well known methods used for the dynamic analysis of multibody systems. These techniques however, are only suitable for a single energy domain systems, e.g. mechanical systems, and can not be used to tackle systems that simultaneously include various physical domains in a unified manner.

The bond graph technique developed since the 1960’s has potential applications in analyzing such complex systems and has been used successfully in many areas [3,4,5,6]. It is a computer oriented method and a pictorial representation of the dynamics of the system, it can also clearly depicts the interaction between elements and model multi-energy domains, for example, the actuator systems, which may be electrical, electro-magnetic, pneumatic, hydraulic or mechanical. Once the bond graph model of the system is ready, the system dynamic equations can be derived from it algorithmically in a systematic manner. This process is usually automated by using appropriated software [3,4,].

Compared with scalar bond graph[3], vector bond graph is more suitable for modelling multibody systems because of its more concise representation manner[7,8,9]. But for multibody systems, the kinematic and geometric constraints between bodies result in differential causality loop, and the nonlinear velocity relationship between the mass center and an arbitrary point on a body leads to the nonlinear junction structure. Current vector bond graph procedures[7,8,9] were found to be very difficult algebraically in derivation of system state space equations automatically on a computer. Besides these, how to model complex multibody systems such as geneva mechanism
or geneva connecting-rod combination mechanism by vector bond graph should be studied further. To solve above problems, a more efficient and practical modelling and simulation procedure for geneva mechanism based on vector bond graph[9] is proposed here.

VECTOR BOND GRAPH MODEL OF POINT-FOLLOWER

With In any multi-body system, the joints impose kinematic constraints on the rigid body elements. A rigid body moving in plane, its revolute joint and translational joint can be modelled by vector bond graph[9]. The point-follower of multibody system is shown in Fig.1, this constraint permits the relative translation of the two body \( B_\alpha \) and \( B_\beta \) along the direction of the slider axis \( \text{I-I} \). Joint point P and Q are fixed on rigid body \( B_\alpha \) and \( B_\beta \) respectively, vector \( h \) is used to describe the relative motion of the two rigid body, \( h = QP = r_\alpha^P - r_\beta^Q \).

Where \( r_\alpha^P \) and \( r_\beta^Q \) represent the position vector of joint point P and Q in global coordinate system \( \text{oxy} \) respectively, \( r_\alpha^P = r_\alpha^\rho + \rho_\alpha \) and \( r_\beta^Q = r_\beta^\rho + \rho_\beta \). So we have

\[
h = r_\alpha^P + \mathbf{A}_\alpha^\rho - r_\beta^Q - \mathbf{A}_\beta^\rho \tag{1}
\]

where \( r_\alpha^\rho \) and \( r_\beta^\rho \) represent the center of mass position vectors of two body \( B_\alpha \) and \( B_\beta \) in global coordinate system \( \text{oxy} \). \( \rho_\alpha^P \) and \( \rho_\beta^Q \) are the position vectors of point P and Q on two body \( B_\alpha \) and \( B_\beta \) in body frame \( c_\alpha^x \alpha^y \) and \( c_\beta^x \beta^y \). \( \theta_\alpha \) and \( \theta_\beta \) are the angular displacement of body \( B_\alpha \) and \( B_\beta \). \( \mathbf{A}_\alpha \) and \( \mathbf{A}_\beta \) are direction cosine matrices.

\[
\mathbf{A}_\alpha = \begin{bmatrix}
\cos(\theta_\alpha) & -\sin(\theta_\alpha) \\
\sin(\theta_\alpha) & \cos(\theta_\alpha)
\end{bmatrix}, \quad \mathbf{A}_\beta = \begin{bmatrix}
\cos(\theta_\beta) & -\sin(\theta_\beta) \\
\sin(\theta_\beta) & \cos(\theta_\beta)
\end{bmatrix}
\]

In Fig.1, \( \mathbf{d}_\alpha \) and \( \mathbf{d}_\beta \) are the unit vectors fixed on rigid body \( B_\alpha \) and \( B_\beta \) in global coordinate system \( \text{oxy} \) respectively, which are parallel to slider axis \( \text{I-I} \). The kinematic constraint condition of point-follower[11] is \( \mathbf{d}_\beta \parallel h \). so we have

\[
\mathbf{d}_\beta \cdot h = 0 \tag{2}
\]
where $d_{\perp}^\beta$ is a unit vector perpendicular to unit vector $d_{\parallel}^\beta$. From Eq.(2), we can get

$$[I A^\beta d_{\perp}^\beta]^T h = 0$$

(3)

where $d_{\parallel}^\beta$ is the unit vector corresponding to $d_{\parallel}^\beta$ in body frame.

$$I = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

From Eq.(3), the constraint equations of point follower can be written as

$$\Phi = [I A^\beta d_{\perp}^\beta]^T (r_a + A^a \rho_a - r_{\perp} - A^\beta \rho_{\perp}^a) = 0$$

(4)

From Eq.(4), the velocity and angular velocity constraint equations of point follower can be written as

$$\dot{\Phi} = (I A^\beta d_{\perp}^\beta)^T \dot{r}_a + [(A^\beta d_{\perp}^\beta)^T (r_a + A^a \rho_a - r_{\perp} - A^\beta \rho_{\perp}^a)] \dot{\theta}_{\perp}$$

$$- (I A^\beta d_{\perp}^\beta)^T \dot{r}_{\perp} + [(A^\beta d_{\perp}^\beta)^T (r_a + A^a \rho_a - r_{\perp} - A^\beta \rho_{\perp}^a)] \dot{\theta}_{\parallel} = 0$$

(5)

where $r_{\perp}$ and $r_a$ are the mass of center velocity vectors of body $B_\perp$ and $B_a$ in the global coordinate system. $\dot{\theta}_{\perp}$ and $\dot{\theta}_a$ are the angular velocities of body $B_\perp$ and $B_a$. From the relations of velocity vectors and angular velocities in Eq.(5), the vector bond graph model of point-follower connecting rigid body $B_a$ and $B_\perp$ undergoing plane motion can be obtained and shown in Fig.2. Where $m_{\perp} = \text{Diag}(m_{cB}, m_{cB}, \beta)$, $m_a = \text{Diag}(m_{cA}, m_{cA}, m_{cA})$, and $m_{cA}$ are the masses of body $B_\perp$ and $B_a$, $J_\perp$ and $J_\parallel$ are the rotational inertias of body $B_\perp$ and $B_a$. The modulus matrices of MTF can be read from Eq.(5) directly

![Fig.2. Vector bond graph model of point-follower](image)

The vector bond graph for the rigid body undergoing planar motion[9] can be coupled to one another satisfying the kinematic constraints[1,2] at the interfaces to get the overall vector bond graph model of planar multibody systems, such as geneva mechanism or geneva connecting-rod combination mechanism. Other energy domain systems, such as electric motor driving system, can be combined with this model to get the complete vector bond graph model of

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more complex multibody systems containing the coupling of multi-energy domains. But the kinematic and geometric constraints result in differential causality. In the derivation of system state space equations, the current vector bond graph procedures[7,8,9] were found to be very difficult algebraically. To eliminate the differential causality, the constraint force vectors at joints can be considered as unknown effort source vectors and added to the corresponding 0-junctions of the system vector bond graph model. For geneva mechanism or geneva connecting-rod combination mechanism, the differential causality can be eliminated completely. Thus, the corresponding algorithm for automatic modeling and simulation based on bond graph theory[10] can be used directly.

In [10], a system bond graph model can be divided into independent energy storage field which consists of I element and C element, dissipative field which consists of R element, source field which consists of Se element and Sf element, and junction structure. From the algebraic relations of input and output vectors in these fields and junction structure, the unified formulae of system state space equations and constraint forces at joints are derived, which are first order ordinary differential equations and easily derived on a computer. For solving the first order ordinary differential equations, the corrected adaptive step size Runge-Kutta method[11] is employed here.

**EXAMPLE SYSTEM**

An external geneva mechanism is shown in Fig.3, where coordinate systems \(o_1x^1y^1\) and \(o_2x^2y^2\) are body frames corresponding to driving plate and external geneva, coordinate system \(oxy\) is global coordinate system, geneva groove number is \(Z\), \(Z=4\). Driving plate radius is \(R\), \(R=80\)mm. Center distance is \(L\), \(L=226.27\)mm. The masses of driving plate and geneva are \(m_1\) and \(m_2\), \(m_1=1.248\)kg, \(m_2=0.565\)kg. The inertias of driving plate and geneva are \(J_1\) and \(J_2\), \(J_1=3.68\times10^{-3}\)kg.m\(^2\), \(J_2=9.35\times10^{-4}\)kg.m\(^2\). The angular displacements of driving plate and geneva are \(q_1\) and \(q_2\), the corresponding angular velocities of driving plate and geneva are \(\omega_1\) and \(\omega_2\), \(\omega_1=120\text{r/min}\).

![Fig.3 Diagram of external geneva mechanism](image)

From the procedure described above, the vector bond graph model of the external geneva mechanism can be made and shown in Fig.4. For simplicity, the process of simulation is only from pin P’s entering to leaving one groove. The constraint force vector of joint which is along the direction of coordinate axis \(O_2X^2\) can be considered as unknown source vector of \(Se_{o_2}\) in Fig.4, and added to the corresponding 0-junctions to eliminate differential causality. Thus, the corresponding algorithm[10] can be used directly.
Inputting the initial values of state variable vector, the physical parameters of the system, and the junction structure matrices[10] into the program associated with the procedure[10] based on MATLAB[11], the system dynamic equations and constraint force equations can be derived and solved automatically, the system responses and the constraint force at joint P are obtained and shown in Fig.5~Fig.8.

For this example, the Newton-Euler method [1,2] was used to determine the corresponding responses of the system, the results are in good agreement with that obtained by the procedure in this paper. However, this process is comparative labor-intensive and tedious. Thus the reliability and efficiency of the modelling and dynamic simulation of complex multibody systems can be improved by the procedure presented here.

![Vector bond graph external geneva mechanism](image1.png)

Fig.4 Vector bond graph external geneva mechanism

![Angular displacement of geneva](image2.png)

Fig.5 The angular displacement of geneva

![Angular velocity of geneva](image3.png)

Fig.6 The angular velocity of geneva
CONCLUSION

The vector bond graph procedure presented here is very suitable for computer aided modeling and simulation of complex systems with the coupling of multi-energy domains. Compared with traditional scalar bond graph method, this vector bond graph procedure is more suitable for complex planar multibody systems because of its more compact and concise representation manner. The method to model geneva mechanism by vector bond graph provides important foundation for automatic modeling and simulation of geneva connecting-rod combination mechanism. Besides these, the differential causalities in the vector bond graph model of geneva mechanism can be avoided by the bond graph augment method proposed here, thus the algebraic difficulties in system automatic modeling and simulation can be overcome, and the constraint forces of joints can be determined directly. By the corresponding algorithm, the automatic modeling and simulation for geneva mechanism can be realized successfully. All these are the necessary supplements to bond graph technique and dynamics of mechanism.

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